

Advanced Growth Model Reduces The Risk of Overvaluing From 'Constant WACC' Assumptions

By Mike Adhikari

"Constant WACC" is an accepted assumption in many business valuations; though it is far from reality for most businesses, or particularly for leveraged transactions. Certainly, the Gordon Growth Model (GGM) is based on constant WACC (and growth) assumptions. And, even when appraisers use multi-period analyses, GGM comes into play when calculating terminal value. The result is that many financial experts correct only for changes in projected growth rates and not for changes in WACC. This often ignored "constant WACC" assumption can cause significant overvaluation.¹

The Advanced Growth Model (AGM) presented here provides a new formula to determine terminal value without the "constant WACC" assumption. AGM is theoretically more correct and is less sensitive to input parameters. It allocates the firm's cash flows among each investor and then discounts each investor's cash flows at his or her respective cost of capital. In AGM, value is the sum of the present values of each investor's cash flows.

AGM is proposed as a replacement to the traditional GGM. GGM works when it is used for discounting single investor cash flows. It also works for multi-investor cash flows when each investor has equal rights to the firm's cash flows. However, today GGM is being widely used for debt and equity valuation even though debt holders have senior rights over equity holders to the firm's cash flows. Unlike GGM, AGM valuation recognizes these uneven rights of debt and equity, and hence provides a theoretically more precise valuation.

Background: Total value in typical DCF analysis is the sum of values in 1) a discrete period analysis in which each year is individually analyzed, and 2) a steady-state (or terminal) phase during which the business is expected to grow at a constant growth rate. Terminal value is the value of the business at the end of the discrete period; or said differently, it is the value of the business at the beginning of the steady-state phase. In the majority of DCF valuations of going-concerns, present values of the cash flows during the discrete period account for a smaller portion of the total value, and the present value of the terminal value accounts for a very large portion, often 60-90% or even more, of the total value. Therefore the accuracy of terminal value is critical to determining the total value of the business.

Today, there are two primary methods to determine terminal value for a going concern. Some analysts (particularly in the transactional world) apply a market multiple to a future benefit stream, such as EBITDA. This method adds subjectivity in selecting the market multiple. Business appraisers are more likely to use the other method, relying on GGM to determine terminal value (or even total value). GGM discounts a firm's future cash flows at a constant WACC,² even when this assumption is not true in leveraged transactions in which debt has to be paid down at a fast pace.³ AGM presented here eliminates the use of WACC. AGM assumes constant return expectations by the debt holder and constant return expectations by the equity holder; but, it does not assume constant blended return,

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Advanced Growth Model

i.e. WACC. AGM thus provides a theoretically more accurate formula for valuation.

WACC can cause overvaluation for a second reason, too. WACC does not differentiate between debt and equity, except for the tax deductibility of debt interest. Ignoring the tax issue, this means that the use of WACC has an implicit assumption that a firm's cash flows will be distributed among debt holders and equity holders without any differentiation between them. In practice, it's rare that checks for debt principal are written at the same time as dividends are paid to the equity holders. Such principal payments cause dividend distributions to the equity holders to be delayed beyond that assumed in valuation using WACC as a discount rate. As a result, actual IRR to equity holders is lower than that used in WACC. Lower than expected equity IRR means overvaluation. (This phenomenon is explained with an example in the article footnoted earlier.)

APV (Adjusted Present Value) method is not a solution to the WACC problem. APV is a different method to get to the same results as WACC. Many academicians have written articles proving that the APV method gives the same results as WACC. So, if WACC ignores debt principal repayment, so does APV. APV clearly spells out that the only value of debt is its tax shield. If we were to assume that tax deductibility of interest is disallowed, then according to APV, debt has no value. This clearly is not true in real life.

The Advanced Growth Model (AGM). In the Advanced Growth Model, debt principal is paid before dividends are paid to equity holders. Hence, the AGM formula requires additional inputs for debt amortization period (p), and the fraction (w_{ad}) of total debt that is amortized. Further, AGM requires input for the holding period (n). Holding period is the time during the steady-state phase when the business is revalued or is deemed to have been sold. The AGM formula assumes that the debt amortization period is more than, or equal to, the holding period ($p \geq n$). The holding period input and the restriction $p \geq n$ helps analyze a stream of cash flows while the

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debt is being paid off, and helps avoid discounting cash flows when debt is fully paid off. As we will see later, the Gordon Growth Model has a holding period of one ($n=1$), meaning that GGM implicitly assumes that the firm is refinanced, or re-leveraged, every year.

New Variables in AGM

1. Debt amortization period (p)
2. % of debt that is amortized (w_{ad})
3. Holding period (n), where $p \geq n$

AGM starts with the EBITDA of a business because EBITDA is generally easier and more reliable to determine. Also, EBITDA is the true value added by the firm; it is the difference between what the market would pay for a firm's products and services and their cost of production before financing costs, taxes and growth re-investment. AGM calculates equity cash flow starting from EBITDA. Also, AGM assumes a C Corp and a stock sale. This allows us to pay taxes at the corporate level and avoid the goodwill tax-shield in an asset sale.⁴

Additional variables described below are standard inputs currently used in finance and valuation:

E_1 – EBITDA in year-1

ΔW_1 – Change in Working Capital in year-1

C_1 – Capital Expenditure in year-1

DA_1 – Depreciation and Amortization in year-1

Z_1 – Unlevered Free Cash Flow to the Firm (same as Net CF to Invested Capital)

X_1 – Excess CF after debt service (same as Net CF to Equity)

w_d – Weight of Debt

w_e – Weight of Equity

t – Tax Rate

r_d – Interest on debt

r_{dt} – After-tax cost of debt

r_e – Cost of Equity

k – After-tax WACC, which is

$$k = w_e r_e + w_d r_{dt}$$

Income Statement

EBITDA	E_1
Less: Interest	$-r_d * D_0$
Less: Dep. and Amort.	$-DA_1$
<hr/>	
Taxable Income	$E_1 - r_d * D_0 - DA_1$
Taxes	$-t * (E_1 - r_d * D_0 - DA_1)$
Net Income	$(1-t) * (E_1 - r_d * D_0 - DA_1)$

Free Cash Flow (Net CF to Invested Capital)

Net Income	$(1-t) * (E_1 - r_d * D_0 - DA_1)$
Plus: After-tax Interest	$(1-t) * r_d$
Plus: Non-cash charges	DA_1
Less: Change in WC	$-\Delta W_1$
Less: Capital Expend.	$-C_1$
<hr/>	
Free Cash Flow (Net CF to Invested Capital)	$Z_1 = (1-t) * E_1 - \Delta W_1 - C_1 - t * DA_1$

Equity Cash Flow (Net Cash Flow to Equity)

Net Income	$(1-t) * (E_1 - r_d * D_0 - DA_1)$
Plus: Non-cash charges	DA_1
Less: Change in WC	$-\Delta W_1$
Less: Capital Expend.	$-C_1$
Less: Debt Service	$-\frac{w_{ad} D_0}{p}$
<hr/>	
Net CF to Equity	$X_1 = (1-t) * (E_1 - r_d * D_0 - DA_1) + DA_1 - \Delta W_1 - C_1 - \frac{w_{ad} D_0}{p}$

The above equation can be simplified into

$$\text{Net CF to Equity } X_1 = Z_1 - r_{dt} * D_0 - \frac{w_{ad} D_0}{p} \dots \dots \dots [1]$$

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Following are standard variables used in deriving the new formula. They are not inputs:

D_0 – Debt at time $t = 0$

E_0 – Equity Value of the firm at time $t = 0$

V_0 – Value of the firm at time $t = 0$

The Advanced Growth Model assumes that the above cash flow to equity is not just “available,” but is distributed to equity holders.⁵ Hence we can discount it and calculate its present value.

Now we will calculate present value (PV) of all equity cash flows (E_0). This equals PV of all X_i and PV of net proceeds to equity at the end of holding period n . Net proceeds to equity at the end of period n equals enterprise value V_n at that time less outstanding debt D_n at that time. Therefore, E_0 is:

$$E_0 = \sum_{i=1}^{i=n} \frac{X_i}{(1+r_e)^i} + \frac{V_n - D_n}{(1+r_e)^n} \dots\dots\dots [2]$$

In the above equation, V_n is the value of the firm at the end of the period n . We are analyzing the value of the firm during its steady-state phase and we are assuming that in the steady-state phase the business is growing at constant rate g and that capital markets are stable. Hence, the enterprise value of the firm will increase in a steady-state phase by the growth rate g . Therefore:

$$V_n = V_0(1+g)^n \dots\dots\dots [3]$$

Equation [3] also means that the price multiple is constant⁶ in steady-state phase, where price multiple (m) is defined as the ratio of Enterprise Value (V) and EBITDA (E); meaning price multiple $m = V/E$.

In equation [2], D_n is the remaining debt at the end of the period n . It is equal to the beginning debt D_0 minus cumulative debt payments during the periods 1 through n . Only w_{ad} % of debt is amortized, and it is amortized over a period of

p years. The remaining debt of $(1-w_{ad})$ % is assumed to be of revolving type with no principal repayment. Therefore, yearly debt repayment is $w_{ad}D_0 / p$ and cumulative debt repayment over n period is $nw_{ad}D_0 / p$. Therefore, D_n is:

$$D_n = D_0 - \frac{nw_{ad}D_0}{p}$$

Substituting V_n and D_n into equation [2] and rearranging we get:

$$E_0 = \sum_{i=1}^{i=n} \frac{X_i}{(1+r_e)^i} + \frac{V_0(1+g)^n}{(1+r_e)^n} - \frac{D_0}{(1+r_e)^n} + \frac{\frac{nw_{ad}}{p} D_0}{(1+r_e)^n} \dots\dots\dots [4]$$

Now, the enterprise value V_0 of the firm is $V_0 = E_0 + D_0$. Substituting the value of E_0 from equation [4], we get enterprise value V_0 as:

$$V_0 = \sum_{i=1}^{i=n} \frac{X_i}{(1+r_e)^i} + \frac{V_0(1+g)^n}{(1+r_e)^n} - \frac{D_0}{(1+r_e)^n} + \frac{\frac{nw_{ad}}{p} D_0}{(1+r_e)^n} + D_0 \dots\dots\dots [5]$$

By substituting the value of X_i from equation [1] into equation [5], and assuming that ΔW_1 , C_1 and DA_1 will grow at rate g , and by simplifying,⁷ we get the new Terminal Value Formula, referred to here as the Advanced Growth Model (AGM) in equation [6]:

$$V_0^{AGM} = \frac{Z_1 \cdot r_e}{r_e - g} \cdot \frac{R_1 - G}{kR_1 - r_e G + w_d(r_e - r_{dt})[1 + \frac{w_{ad}}{p} \cdot (R_2 - n)]} \dots\dots\dots [6]$$

Advanced Growth Model

Where,

$$R_1 = (1 + r_e)^n$$

$$G = (1 + g)^n$$

$$R_2 = (R_1 - 1) / r_e$$

k = Conventional after-tax WACC

w_{ad} = % amortized debt

p = Debt Amortization period

n = Holding period

and

Z_1 = Unlevered FCF a.k.a. NCF_{ic}

$$Z_1 = (1 - t)E_1 - \Delta W_1 - C_1 + t.DA_1$$

where

E_1 = EBITDA

ΔW_1 = Change in Working Capital

C_1 = Capital Expenditure

DA_1 = Depreciation and Amortization

GGM is a special case of AGM. AGM is a general formula. AGM becomes GGM if we apply GGM assumptions. For $n = 1$, the AGM formula collapses to GGM. Holding period of one ($n = 1$) implies continuous refinancing or continuous re-leveraging. This is the assumption built into GGM and hence AGM collapses to GGM when $n = 1$.

AGM does not collapse to GGM if the debt principal is never repaid (Debt amortization period $p =$ infinite, or $w_{ad} = 0\%$, meaning that 0% of debt is amortized, will result in no debt amortization). However, AGM does collapse to GGM if we assume, in addition to no debt amortization, that growth, $g = 0$. AGM does not collapse to GGM when $g > 0$ because GGM assumes that a business will borrow more as it grows and that such new debt will be distributed to equity holders. AGM does not make this assumption. Hence, AGM does not collapse to GGM when debt is not amortized and $g > 0$.

AGM vs. GGM comparison. We will now compare results of AGM with the results of GGM. All things being equal, GGM valuation will be higher than AGM valuation because equity cash

flows that are implicit in GGM do not materialize to repay debt principal.

AGM does not have the simplicity and the elegance of GGM. Prior to the spreadsheet era, it would have been difficult to use AGM. However, with spreadsheet programs, it is now infinitely easier to use AGM.

Gordon Growth Model

$$V_0^{GGM} = \frac{Z_1}{(k - g)}$$

Advanced Growth Model

$$V_0^{AGM} = \frac{Z_1 \cdot r_e}{r_e - g} \cdot \frac{R_1 - G}{kR_1 - r_e G + w_d(r_e - r_{dt})[1 + \frac{w_{ad}}{p} \cdot (R_2 - n)]}$$

$$R_1 = (1 + r_e)^n$$

$$G = (1 + g)^n$$

$$R_2 = (R_1 - 1) / r_e$$

k = Conventional after-tax WACC

w_{ad} = % amortized debt

p = Debt Amortization period

n = Holding period

Table 1 shows comparison of AGM vs. GGM results. In the results, the price multiple is the ratio of V_0/E_0 , where E_0 is year-0 EBITDA. E_0 is assumed to grow at rate g in year-1 to E_1 , so $E_1 = E_0 \cdot (1 + g)$. Table 1 also assumes no capital expenditures, no change in working capital, and no depreciation and amortization. Hence,

$$Z_1 = (1 - t) \cdot E_1 = (1 - t) \cdot (1 + g) \cdot E_0.$$

Following are a few observations of using AGM vs. GGM:

1. All things being equal GGM overvalues a business.
2. Higher spread between cost of equity and cost of debt, $r_e - r_d$, means higher GGM overvaluation.
3. Higher taxes mean higher GGM overvaluation.

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- Higher growth means higher GGM overvaluation.
 - Higher leverage means higher GGM overvaluation.
 - Shorter amortization period means higher GGM overvaluation.
 - Higher % of amortized debt means higher GGM overvaluation.
 - A longer holding period means higher GGM overvaluation.
 - AGM assumes that excess CF will be paid to equity. Instead, if excess CF is used to prepay debt, GGM overvaluation will be even higher.⁸
 - AGM is less sensitive to input parameters than GGM.
 - AGM is significantly less sensitive to $(k - g)$ than GGM.
- "WACC as used in capitalization formula causes overvaluation" by Mike Adhikari, October 2003 *BVU*, www.BVResources.com.
 - Some people use GGM to value equity. This can be done as long as there is no debt. In this situation, however, the value could be low because of the high cost of equity.
 - From a valuation perspective, a business would have a high leverage even when determining terminal value. Value is determined by what a willing buyer would pay and what a willing seller would accept. A seller wants the highest price, which means a buyer would be deemed to maximize leverage even when determining terminal value.
 - AGM variations for C Corp vs. S Corp, Asset vs. Stock sale can best be handled through spreadsheet modeling.
 - In LBO transactions, lenders often restrict such excess cash flow distribution to equity holders. I have tried to develop a valuation formula under that assumption, but it is too complex. Such variation can be best handled through spreadsheet modeling.
 - $m_n = V_n/E_n$. In a steady-state phase $V_n = V_0 \cdot (1+g)^n$ and, $E_n = E_0 \cdot (1+g)^n$. Therefore, $m_n = V_0 \cdot (1+g)^n / E_0 \cdot (1+g)^n$. Simplifying, $m_n = V_0/E_0$, which is m_0 . Therefore, $m_n = m_0$, meaning price multiples are constant in a stable-phase.
 - Simplification takes 10 pages of algebraic equations.
 - In the previously referenced article at BVResources, GGM overvaluation was 25% for the scenario of 50% debt, no growth and no taxes. GGM overvaluation is only 14.9% using AGM for the same scenario. This is because excess CF was used to pre-pay debt in the previous article, whereas in AGM, such excess CF is distributed to equity holders.

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	Traditional			Input Variables		n	New		Output Price Multiple $= V_0/E_0$, where $E_1 = E_0 \cdot (1+g)$		
	r_e	r_d	w_d	g	t		p	w_{ad}	GGM	AGM	Overvaluation
								$p \geq n$			
	30%	10%	50%	0%	0%	5	5	100%	5.00	4.351	14.9%
	30%	10%	50%	0%	40%	5	5	100%	3.33	2.781	19.9%
	30%	10%	50%	5%	40%	5	5	100%	4.85	3.666	32.2%
	30%	10%	75%	0%	0%	5	5	100%	6.67	5.136	29.8%
	30%	10%	75%	0%	40%	5	5	100%	5.00	3.455	44.7%
	30%	10%	75%	5%	40%	5	5	100%	9.00	4.745	89.7%

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